

Homework 8

Wouter Fransen
w.fransen@students.uu.nl

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Remark: Recall that a sheaf \mathcal{F} on a space X is flabby if for every $U \subseteq V$, the restriction $\mathcal{F}(V) \rightarrow \mathcal{F}(U)$ is surjective. (Or equivalently, every $\mathcal{F}(X) \rightarrow \mathcal{F}(U)$ is surjective)

Exercise 1: Suppose

$$0 \longrightarrow \mathcal{F} \longrightarrow \mathcal{G} \longrightarrow \mathcal{H} \longrightarrow 0$$

is an exact sequence of sheaves.

1. Show that if \mathcal{F} is flabby, then

$$0 \longrightarrow \mathcal{F}(U) \longrightarrow \mathcal{G}(U) \longrightarrow \mathcal{H}(U) \longrightarrow 0$$

is exact for every $U \subseteq X$ open.

2. Show that if \mathcal{F} and \mathcal{G} are flabby, then \mathcal{H} is flabby.

Hint for 1: For surjectivity, fix $s \in \mathcal{H}(U)$ and consider the set of pairs (t, V) where $V \subseteq U$ open and $s|_V = t$. Then use Zorn's Lemma.