Pro-fundamental group of a topos

Deik van der Horst, d.s.vanderhorst@students.uu.nl, 5985137.

1. The Fundamental Group in Topology

This lecture will be about the pro-fundamental group of a topos. We give a short overview of the fundamental group $\pi_1(X, x)$ of a topological space X. A common way of describing $\pi_1(X, x)$ is as the group of loops at the fixed point $x \in X$ up to homotopy. Note that having an interval object is crucial for working with loops, when working with a topos this does not function well. Therefore we will characterize $\pi_1(X, x)$ in the following way.

We recall from topology that a covering space for X is a space $\tilde{X}$ and a morphism $p : \tilde{X} \to X$ such that any $x \in X$ has an open neighborhood U for which $p^{-1}(U)$ is a disjoint union of open sets in $\tilde{X}$, such that each open set is projected homeomorphically onto U by p.

When a topological space is nice enough, in particular locally pathconnected semilocally simply connected, we get a correspondence between subgroups of the fundamental group and covering spaces of X.

We remark that something similar happens with Galois extensions L/K and the Galois group $\mathrm{Gal}(L/K)$ in field theory: intermediate field extensions correspond to subgroups of $\mathrm{Gal}(L/K)$.

Recall that locally constant sheaves correspond to covering spaces (consider sections to p), so the fundamental group of a topological space can be seen as the group classifying locally constant sheaves on X. This will be the idea behind our definition of the pro-fundamental group of a topos.

The pro-fundamental group of a topos can be seen as a generalization in two directions. Firstly, it can be seen as a rephrasing of the fundamental group of a topos in the more general topos-theoretic language. Secondly, we can view it as a generalization of the fundamental group of nice topological spaces to the fundamental group of spaces that are not nice. For such a topological space, we cannot find a fundamental group, but we can find a weaker kind of object, a pro-group.

2. Ind- and Pro-Objects

We now introduce pro-objects, the pro-fundamental group of a topos will be a pro-object in the category of groups.

We will first discuss ind-objects; ind-objects are the dual notion to pro-objects. We will see that the treatment of ind-objects is easier that that of pro-objects, so obtaining pro-objects by dualizing the construction of ind-objects will be our approach.

Let $\mathcal{C}$ be a small category, and $y : \mathcal{C} \to \mathrm{Psh}(\mathcal{C})$ the Yoneda embedding. Recall the universal property of the presheaf topos $\mathrm{Psh}(\mathcal{C})$: For any cocomplete category $\mathcal{D}$ and functor $\mathcal{C} \to \mathcal{D}$, there is a functor colimit preserving functor $\mathrm{Psh}(\mathcal{C}) \to \mathcal{D}$, unique up to unique isomorphism, such that the following diagram commutes up to isomorphism:

$$\begin{tikzcd} \mathcal{C} \arrow[r, "y"] \arrow[d, "F"] & \mathrm{Psh}(\mathcal{C}) \arrow[dl, "\exists! \bar{F}"] \\ \mathcal{D} & \end{tikzcd}$$

That is, $\mathrm{Psh}(\mathcal{C})$ is the free colimit completion of $\mathcal{C}$. We could also give a different description of the free colimit completion of $\mathcal{C}$. Recall that we can view any presheaf F as the colimit of representables, so $F = \mathrm{colim}_{i\in I}\, y(x_i)$. Therefore, we could also define $\mathrm{Psh}(\mathcal{C})$ as the category containing all $F = \mathrm{colim}_{i\in I}\, y(x_i)$ for any diagram I.

Suppose that instead of considering the general free colimit completion of $\mathcal{C}$, we are interested in free *filtered* colimit completion of $\mathcal{C}$.

Definition 1 (filtered diagram)**.** *A category I is filtered if the following holds:*

- *I is nonempty,*
- *For any $i, j \in I$ there is $k \in I$ such that $i \to k \leftarrow j$,*
- *For each parallel pair $a, b : i \to j$ there is an arrow $j \to k$ equalizing the pair.*

A presheaf F is filtered when we can write $F = \mathrm{colim}_{i\in I}\, y(x_i)$, for I filtered.

Proposition 2. *Filtered colimits commute with finite limits in* Set*.*

Note that this implies that cofiltered limits commute with finite colimits in any presheaf topos, and hence in any Grothendieck topos (a Grothendieck topos is a reflexive subcategory of the presheaf topos on the site).

We make the following remark: $F = \mathrm{colim}_{i\in I}\, y(x_i)$ if filtered if and only if the category $\mathrm{Elts}(F) \cong (y \downarrow F)$ is filtered. To sketch the proof of this: If F is filtered, then I is equivalent to the category of elements which is filtered. If $\mathrm{Elts}(F)$ is filtered, then we can write $F \cong \lim_{(x,s)\in \mathrm{Elts}(F)} y(x)$ making F a filtered colimit of representables.

Definition 3 ($\mathrm{Ind}(\mathcal{C})$)**.** *We define the category Ind($\mathcal{C}$) in the following way:*

- *An Ind-object of $\mathcal{C}$ is a filtered diagram $I \to \mathcal{C}$.*
- *We define morphisms in $\mathrm{Ind}(\mathcal{C})$ as*

$$\mathrm{Hom}_{\mathrm{Ind}(\mathcal{C})}(\mathrm{colim}_{i\in I}\, x_i, \mathrm{colim}_{j\in J}\, z_j) \cong \lim_{i\in I} \mathrm{Hom}_{\mathcal{C}}(x_i, \mathrm{colim}_{j\in J}\, z_j)$$
$$\cong \lim_{i\in I} \mathrm{colim}_{j\in J} \mathrm{Hom}_{\mathcal{C}}(x_i, z_j)$$

The first step here follows because the contravariant Hom takes colimits to limits in the first variable, the second step follows from the Yoneda lemma.

We could also view $\mathrm{Ind}(\mathcal{C})$ as the full subcategory of $\mathrm{Psh}(\mathcal{C})$ consisting of presheaves F that can be written as a filtered colimit of representables.

Note that the Yoneda embedding restricts to $\mathcal{C} \xrightarrow{y} \mathrm{Ind}(\mathcal{C})$. We get the following universal property of $\mathrm{Ind}(\mathcal{C})$: If a category $\mathcal{D}$ has filtered colimits and $F : \mathcal{C} \to \mathcal{D}$ preserves filtered colimits, then there is a functor $\bar{F}$, unique up to unique isomorphism, such that the following commutes up to isomorphism:

$$\begin{tikzcd} \mathcal{C} \arrow[r, "y"] \arrow[d, "F"'] & \mathrm{Ind}(\mathcal{C}) \arrow[dl, "\exists! \bar{F}"] \\ \mathcal{D} & \end{tikzcd}$$

We end our discussion of Ind-objects with an example.

Remark 4. *Let* Set_{fin} *be the category of finite sets. We show that the category* $\mathrm{Ind}(\mathrm{Set}_{fin})$ *is equivalent to* Set. *Take any filtered diagram* $I \to \mathrm{Set}_{fin}$. *Note that we can write any set as the colimit of finite sets, and a morphism in* $\mathrm{Ind}(\mathrm{Set}_{fin})$ *induces a morphism between the colimits. This gives the equivalence.*

We move to pro-objects. By dualizing the definition of filtered, we get a notion of being cofiltered.

Definition 5. *A category I is cofiltered if the following holds:*

- *I is nonempty,*
- *For any $i, j \in I$ there is $k \in I$ such that $i \leftarrow k \to j$,*
- *For each parallel pair $a, b : i \to j$ there is an arrow $k \to i$ equalizing the pair.*

Now $F \in \mathrm{Psh}(\mathcal{C}^{op})^{op}$ is cofiltered in a category $\mathcal{C}$ when we can write $F \cong \lim_{i \in I} k(x_i)$, for I cofiltered.

Note that when a category I^{op} is cofiltered, then I is filtered. When F is a cofiltered limit of representables by k in $(\mathrm{Psh}(\mathcal{C}^{op}))^{op}$, then F is a filtered colimit of representables by y in $\mathrm{Psh}(\mathcal{C})$.

We can now rephrase Proposition 2 as follows:

Proposition 6. *Cofiltered limits commute with finite colimits in* Set^{op}.

We have constructed $\mathrm{Ind}(\mathcal{C})$ as the free filtered colimit completion of $\mathcal{C}$; we want to construct $\mathbf{Pro}(\mathcal{C})$ as the free cofiltered limit completion of $\mathcal{C}$. We define

$$\mathbf{Pro}(\mathcal{C}) := (\mathrm{Ind}(\mathcal{C}^{op}))^{op}.$$

When defining $\mathrm{Ind}(\mathcal{C})$, we used the contravariant Yoneda embedding $y : \mathcal{C} \to \mathrm{Psh}(\mathcal{C}), X \mapsto \mathrm{Hom}(,X)$. Because we are dualizing the construction, we now need to use the covariant $k : \mathcal{C} \to (\mathrm{Psh}(\mathcal{C}^{op}))^{op} \cong \mathrm{Hom}(\mathcal{C}^{op}, \mathrm{Set}^{op})$.

Note that Set^{op} is very different from Set, for example it is not a topos. We remark that Set^{op} is equivalent to the category of complete atomic boolean algebras (power sets).

Now we want $\mathbf{Pro}(\mathcal{C})$ to be the full subcategory of $(\mathrm{Psh}(\mathcal{C}^{op}))^{op}$ consisting of presheaves $F \cong \lim_{i \in I} k(x_i)$ that are a cofiltered limit of a representable for the covariant Yoneda embedding.

Phrased differently: an object of $\mathbf{Pro}(\mathcal{C})$ is a cofiltered diagram $I \to \mathcal{C}$. A morphism of $\mathbf{Pro}(\mathcal{C})$ is defined as

$$\mathrm{Hom}_{\mathbf{Pro}(\mathcal{C})}((X_i)_{i\in I}, (Y_j)_{j \in J}) \cong \mathrm{Hom}_{(\mathrm{Psh}(\mathcal{C}^{op}))^{op}}(\lim_i X_i, \lim_j Y_j)$$

$$\cong \lim_j \mathrm{colim}_i \, \mathrm{Hom}_{\mathcal{C}}(X_i, Y_j).$$

We can also express the situation with $\mathbf{Pro}(\mathcal{C})$ as a universal property. Let $F : \mathcal{C} \to \mathcal{D}$ be a functor into a category $\mathcal{D}$ with cofiltered limits, then there is a functor $\bar{F}$, unique up to unique isomorphism, making the following commute up to isomorphism:

$$\begin{tikzcd} \mathcal{C} \arrow[r, "k"] \arrow[dr, "F"'] & \mathbf{Pro}(\mathcal{C}) \arrow[d, "\bar{F}"] \\ & \mathcal{D} \end{tikzcd}$$

Recall that $X \in \mathcal{C}$ represents a functor F when $F \cong \mathrm{Hom}_{\mathcal{C}}(-, X)$.

Definition 7 (Pro-representable)**.** *A functor F is pro-representable by a pro-object X if $F \cong \mathrm{Hom}_{\mathbf{Pro}(\mathcal{C})}(-, X)$.*

Note that any representable functor is in particular pro-representable. That is, we have inclusions

$$\mathcal{C} \to \mathbf{Pro}(\mathcal{C}) \to \mathrm{Psh}(\mathcal{C}).$$

Remark 8. *We show that $\mathbf{Pro}(\mathrm{Set}_{fin})$ is equivalent to the category Top_{TDCH} of totally disconnected compact Hausdorff topological spaces. Note that we have a functor $disc : \mathrm{Set}_{fin} \to \mathrm{Top}$ which equips a finite set with the discrete topology. We know that* Top *is complete, so we get a cofiltered limit preserving functor $\overline{disc} : \mathbf{Pro}(\mathrm{Set}_{fin}) \to \mathrm{Top}$. Checking that the essential image of $\overline{disc}$ corresponds to Top_{TDCH}, and that $\overline{disc}$ gives an equivalence between Top_{TDCH} and $\mathbf{Pro}(\mathrm{Set}_{fin})$ is left as an exercise.*

A short remark: an alternative approach to using pro-groups is the use of locales. A locale is a generalization of a topological space, in which we can take the limit of a progroup without losing information. We can say: a progroup is a group object in the category of locales. A lot of the literature on progroups is written in the language of locales.

3. The Classifying Topos of a Pro-Group

Definition 9 (Projective system). *A collection of groups $\mathcal{G} = \{G_i\}_{i \in I}$, with I a directed poset (now I is cofiltered when viewed as a category), is called a projective system when we have maps $f_{ij} : G_j \to G_i$ for all $i \in I$ satisfying the following conditions:*

1. $f_{ii} = \mathrm{id}_{G_i}$ *for all* $i \in I$,
2. $f_{ik} = f_{ij} f_{jk}$ *for all* $i \leq j \leq k$.

Note that $\mathcal{G}$ forms a pro-group.

When all f_{ij} are surjective, we call $\mathcal{G}$ a strict projective system.

We remark that by taking I a directed poset, instead of any cofiltered category, we do not lose any generality. It is a fact that every cofiltered category admits a final functor from a directed set (recall that a functor F is final if restricting diagrams along F does not change their colimit). Hence we can replace a cofiltered category with a directed set.

Take some set E. We can define a left-action of $\mathcal{G}$ on E in the following way. First we take a family of sets $(E_i)_i \subseteq E$ such that $\bigcup_i E_i = E$. We now take for each i an action $\rho_i : G_i \times E_i \to E_i$ of G_i on the set E_i. For this action, we demand the following: for $j \geq i$, the set of fixed points of the kernel of the group homomorphism $G_j \xrightarrow{f_{ij}} G_i$ is the set $E_i \subseteq E_j$. That is, $\mathrm{Fix}(\mathrm{Ker}(f_{ij})) = E_i$.

Sets equipped with a left $\mathcal{G}$-action form a category, the *classifying topos* of $\mathcal{G}$ which we will denote $B\mathcal{G}$, in the following way. Objects are pairs $(E, (E_i)_i)$ with $\bigcup_i E_i = E$ and for each i a left action of G_i on E_i. A morphism $f : (E, (E_i)_i) \to (F, (F_i)_i)$ consists of a map $f : E \to F$ such that for every $i \in I$, $f(E_i) \subseteq F_i$, and that for all $i, x \in E_i, g \in G_i$ we have $f(x \cdot g) = g \cdot f(x)$.

By applying Girauds criterion, we see that $B\mathcal{G}$ is in fact a topos.

Remark 10. *Suppose that we have a strict projective system of groups $\mathcal{G} = (G_i)_i$ in which all groups G_i are finite. We can get a concrete handle on what the resulting progroup (a profinite group) looks like in this case: $\mathcal{G}$ will be a topological group which is compact Hausdorff totally disconnected.*

In the following, we will define an appropriate strict projective system of groups $\mathcal{G}$ such that the classifying topos $B\mathcal{G}$ is the quotient topos of locally constant objects in $\mathcal{E}$; recall that in the topological case, a locally constant sheaf corresponds to a covering space over X. We will then define the pro-fundamental group of $\mathcal{E}$ to be $\mathcal{G}$.

4. Locally Constant Objects

Let $\mathcal{E}$ be a topos with terminal object 1. We take $\mathcal{S}$ to be a point topos, that is a topos isomorphic to Set. Recall that we have a unique geometric morphism

$$\gamma : \mathcal{E} \to \mathcal{S},$$

with $\gamma^* : I \to \coprod_{i \in I} 1$ called the constant object functor, and $\gamma_* : X \mapsto \Gamma_{\mathcal{E}}(X) := \mathrm{Hom}_{\mathcal{E}}(1, X)$ the global sections functor.

Note that for γ to be unique, we only need to show that γ^* is unique. We know that γ^* takes the terminal object in $\mathcal{E}$ to the singleton and γ commutes with colimits, every set is a colimit of the singleton, and thus γ is unique.

We recall the following connectedness definitions for a topos.

Definition 11 (Cover of a topos)**.** *A family $\{f_i : U_i \to U\}_{i \in I}$ is called epimorphic when for any two morphisms g, h with domain U we have that $(\forall i \in I)(gf_i = hf_i)$ implies $g = h$.*

A cover for a topos is an epimorphic family $\{U_i \to 1\}_{i \in I}$.

Note that in a topos (or more generally, in a category with I-coproducts), a family $\{f_i : U_i \to U\}_{i \in I}$ is epimorphic if and only if the canonical morphism $\coprod_{i \in I} U_i \to U$ is an epimorphism. Note that considering the family of morphisms instead of the coproduct has the advantage that the coproduct may not exist in the underlying site, but the morphisms in the family do exist in the site. For notational simplicity, we will in the following also consider a cover as some epimorphism $U \to 1$.

Definition 12 (Locally connected)**.** *We call $\mathcal{E}$ locally connected if the inverse image functor γ^* has a left adjoint, which we will denote by $\gamma_!$.*

Definition 13 ((Semi-locally) connected)**.** *We call $\mathcal{E}$ connected if the inverse image functor γ^* is fully faithful.*

We call $\mathcal{E}$ semi-locally connected when there is a cover $\{U_i\}_{i \in I}$ of $\mathcal{E}$ such that $\mathcal{E}/U_i$ is a connected topos for all $i \in I$.

It can be shown that for $\mathcal{E}$ the topos of sheaves on a topological space X, $\mathcal{E}$ is connected as a topos if and only if X is connected as a topological space.

Proposition 14. *Suppose $\mathcal{E}$ is locally connected, so we have $\gamma_! \dashv \gamma^*$. Now $\mathcal{E}$ is connected if and only if $\gamma_!$ preserves the terminal object.*

Proof. From the adjunction $\gamma_! \dashv \gamma^*$, we get the counit $\epsilon : \gamma_! \circ \gamma^* \to \mathrm{id}_{\mathcal{S}}$. Suppose γ^* is fully faithful, then ϵ is an isomorphism and $\gamma_!(1) \cong \gamma_!(\gamma^*(1)) \cong 1$.

Now suppose that $\gamma_!$ preserves 1. Now we have

$$\gamma_!(\gamma^*(A)) \cong \gamma_!(\gamma^*(\coprod_{a \in A} 1)) \cong \coprod_{a \in A} \gamma_!(\gamma^*(1)) \cong \coprod_{a \in A} 1 \cong A$$

so ϵ is an isomorphism and γ^* is fully faithful. $\square$

Remark 15. *In these definitions, we could replace the point topos $\mathcal{S}$ by some other topos $\mathcal{F}$, giving rise to the notion of $\mathcal{F}$-connectedness of a topos.*

We now give a definition of locally constant objects in a general topos.

Definition 16 ($\mathcal{U}$-Split)**.** *Take X an object of a topos $\mathcal{E}$, and let $\mathcal{U} = \{U_i\}_{i\in I}$ be a cover of $\mathcal{E}$. We say that X is $\mathcal{U}$-split, when there exists a family of sets $\{S_i\}_{i\in I}$, such that there is an isomorphism*

$$\theta_i : \gamma^* S_i \times U_i \to X \times U_i$$

in $\mathcal{E}/U_i$ for each $i \in I$.

We denote the collection of all $\mathcal{U}-$split object by $\mathrm{Split}(\mathcal{U}) := \{X \mid X \;\; \mathcal{U}\text{-split}\}$, and let $\mathrm{Split}(\mathcal{E})$ denote the union $\bigcup_{\mathcal{U}\text{ a cover}} \mathrm{Split}(\mathcal{U})$.

Definition 17 (Locally constant object)**.** *We call an object $X \in \mathcal{E}$ locally constant, if X is $\mathcal{U}-$split for some cover $\mathcal{U}$.*

We call X (globally) constant when $X \cong \gamma^*(S)$ for some set S.

Using locally constant objects, we can say when a topos is simply connected.

Definition 18 (Simply connected topos)**.** *We call a topos $\mathcal{E}$ simply connected if $\mathcal{E}$ is connected, and any locally constant object of $\mathcal{E}$ is globally constant.*

We call $\mathcal{E}$ locally simply connected when there is a cover $\{U_i\}_{i\in I}$ of $\mathcal{E}$ such that $\mathcal{E}/U_i$ is a simply connected topos for all $i \in I$.

5. The Topos of Locally Constant Objects

In the following, we assume $\mathcal{E}$ to be connected locally connected. Some terminology: we call $\mathcal{F} \subset \mathcal{E}$ a quotient topos of $\mathcal{E}$ when the inclusion is the inverse image of a geometric morphism $\mathcal{E} \to \mathcal{F}$ (and thus preserves finite limits).

Proposition 19. *For any cover U of $\mathcal{E}$, $\mathrm{Split}(U) \subset \mathcal{E}$ is a quotient topos with inverse image $\mathcal{E} \to \mathrm{Split}(U)$ given by the inclusion.*

Moreover, $\mathrm{Split}(U)$ is a boolean topos, that is, any object of the topos has a complement.

We remark that $\mathrm{Split}(U)$ is connected, whenever $\mathcal{E}$ is.

We collect all locally constant objects in the set

$$GLC(\mathcal{E}) := \{X \in \mathcal{E} \mid X \text{ locally constant}\}.$$

Proposition 20. *$GLC(\mathcal{E}) \subset \mathcal{E}$ is a topos, with as inverse image the inclusion.*

As we will see, $GLC(\mathcal{E})$ is in fact a so-called Galois topos, We will now describe how to construct $GLC(\mathcal{E})$ as a certain limit.

Let $\mathrm{Cov}(\mathcal{E})$ be the set containing all coverings U of $\mathcal{E}$. We can equip $\mathrm{Cov}(\mathcal{E})$ with a refinement partial order in the following way:

$$U \leq V \text{ if and only if there is a map } U \to V.$$

It is an easy exercise to show that $\mathrm{Cov}(\mathcal{E})$ is in fact a directed poset.

We now get the following diagram:

$$\mathrm{Split}(U_i) \leftarrow \mathrm{Split}(U_j) \leftarrow ... \leftarrow GLC(\mathcal{E}) \leftarrow \mathcal{E}.$$

Now the topos $GLC(\mathcal{E})$ arises as the inverse limit of toposes indexed by the directed poset $\mathrm{Cov}(\mathcal{E})$.

6. The Pro-Fundamental Group of a Topos

As next step towards defining the pro-fundamental group of $\mathcal{E}$, we define Galois toposes. From now on, we also suppose that $\mathcal{E}$ is pointed, so we have a geometric morphism $f : \mathcal{S} \to \mathcal{E}$.

For any object $X \in \mathcal{E}$, we can consider its automorphism group $\mathrm{Aut}(X)$.

Definition 21 (Galois object). *An object $A \in \mathcal{E}$ is a Galois object, when it is non-empty, connected, and it is an $\mathrm{Aut}(A)-$torsor.*

This definition unfolds as follows. An object $A \in \mathcal{E}$ is an $\mathrm{Aut}(A)-$torsor, when $A \times \gamma^*(\mathrm{Aut}(A)) \to A \times A, (a, \varphi) \mapsto (a, \varphi(a))$, is an isomorphism. An object A is non-empty when it is not the initial object, and A is connected when $A \to 1$ is an epimorphism.

Definition 22 (Galois topos). *A topos is generated by an object A if for every $X \in \mathcal{E}$, there is an epimorphism $A \to X$.*

A Galois topos is a connected locally connected topos generated by its Galois objects.

Proposition 23. *For any cover U of $\mathcal{E}$, $\mathrm{Split}(U)$ is a Galois topos, and*

$$\mathrm{Split}(U) \cong B\,\mathrm{Aut}(A)^{op}$$

for some Galois object $A \in \mathcal{E}$ representing the point (so $f^* \cong [A, -] : \mathrm{Split}(U) \to \mathrm{Set}$).

We have characterized $GLC(\mathcal{E})$ via the diagram

$$\mathrm{Split}(U_i) \leftarrow \mathrm{Split}(U_j) \leftarrow ... \leftarrow GLC(\mathcal{E}) \leftarrow \mathcal{E},$$

which we can now write as

$$B\mathrm{Aut}(A_i)^{op} \leftarrow B\mathrm{Aut}(A_j)^{op} \leftarrow ... \leftarrow GLC(\mathcal{E}) \leftarrow \mathcal{E}.$$

Hence, $\lim_i B\mathrm{Aut}(A_i)^{op} = GLC(\mathcal{E})$.

We now use the following theorem by Moerdijk:

Theorem 24 (Inverse Limit Theorem)**.** *Let* $\{G_i\}_i$ *be an inverse filtered system of groups. Now*

$$\lim_i BG_i \cong B \lim_i G_i.$$

Using this theorem, we get $B \lim_i \mathrm{Aut}(A_i)^{op} = GLC(\mathcal{E})$. We now define $\pi_1(\mathcal{E}, f)$ to be the pro-group $\lim_i \mathrm{Aut}(A_i)^{op}$.

7. Optional Homework

(1.) Finish the proof of Remark 8.

(2.) Let X be a topological space, and $Sh(X)$ the topos of sheaves on X. Show that X is connected as topological space, if and only if $Sh(X)$ is connected as topos.

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