HOMEWORK FOR THE COURSE FOUNDATIONS OF MATHEMATICS
2025-2026

LÉONARD GUETTA

1. Homework 1 [Deadline: November 20, 2025, 23:59]

Homework longer than 3 pages long will not be accepted!

(1) Let $f\colon X \to Y$ be a injective function.
   (a) (2 pts) Prove without the axiom of choice that if X is non-empty, then there exists a surjection $g\colon Y \to X$ such that $g(f(x)) = x$ for every $x \in X$.
   (b) (1 pt) What happens when $X = \emptyset$? When can you define g as above?

(2) (3 pts) Prove that
$$|\mathbb{N}^{\mathbb{N}}| = 2^{|\mathbb{N}|}.$$

(3) (4 pts) Let X be an infinite set. Prove that
$$|X| = |X| + 1.$$
Hint: Proposition 1.2.2 of the textbook.